Problem 1

Consider the decays $\mu^+ \to e^+ + \nu_e + \bar{\nu}_e$ and $\tau^+ \to e^+ + \nu_e + \bar{\nu}_\tau$. The branching ratios are 100% for the first, 16% for the second. The $\mu$ lifetime is 2.2 $\mu$s. Calculate the $\tau$ lifetime. You can use the equation relating the partial width to the mass of the parent lepton with $\epsilon = 0$ (that is, ignore corrections due to the electron mass); in the case of the $\tau$, the branching ratio can be used to derive the total width from the partial width.

Problem 2

What is the minimum momentum of the electron from a muon decay at rest? What is the maximum? (Before doing any calculations, draw the two arrangements of final-state momenta in the muon rest frame that you think should give the minimum and maximum electron momentum.)

Problem 3

Consider the neutrino cross section on an electron, $\sigma(\nu_\mu + e^- \to \nu_\mu + e^-) \approx \frac{G_F^2}{\pi} s$ and on an “average nucleon” (namely, the average between the cross sections on a proton and a neutron) $\sigma(\nu_\mu + N \to \mu^- + h) \approx 0.2 \frac{G_F^2}{\pi} s$ at energies $\sqrt{s} \gg m$, where $m$ is the target mass and $h$ any hadronic final state (sometimes written as $X$, to imply that it is not identified); the factor 0.2 is related to the quark distributions inside the nucleon. Calculate the ratio of these two cross sections at a neutrino energy $E_\nu = 50$ GeV (the target, electron or nucleon, is at rest). How does this ratio depend on beam energy? Calculate $\sigma/E_\nu$ for the two reactions.

Problem 4

Consider the measured decay rates $\Gamma(D^+ \to \bar{K}^0 + e^+ + \nu_e) = (7 \pm 1) \times 10^{10}$ s$^{-1}$ and $\Gamma(\mu^+ \to e^+ + \bar{\nu}_\mu + \nu_e) = (2.2 \mu s)^{-1}$. Justify the very large ratio for these two quantities. Hint: Estimate the ratio of the widths of the two decays by approximating $D^+ \to \bar{K}^0 + e^+ + \nu_e$ as the decay of a free $c$ quark into a free $s$ quark, $c \to s + e^+ + \nu_e$, so that you can use the same
equation as for lepton decay; but also consider what else is different between that process and the muon decay.

**Problem 5**

Consider the decays

1. \( D^+ \rightarrow \bar{K}^0 + \pi^+ \)
2. \( D^+ \rightarrow \bar{K}^0 + K^+ \)
3. \( D^+ \rightarrow \pi^0 + K^+ \).

For each of them find the valence quark composition and establish whether it is favored, suppressed, or doubly suppressed. (No calculations are needed here.)

**Problem 6**

Consider the experimental values of the branching ratio \( \frac{\Gamma (\Sigma^- \rightarrow n + e^- + \bar{\nu}_e)}{\Gamma_{\text{tot}}} \approx 10^{-3} \) and of the upper limit \( \frac{\Gamma (\Sigma^+ \rightarrow n + e^+ + \nu_e)}{\Gamma_{\text{tot}}} < 5 \times 10^{-6} \). Give the reason for such a difference. (Again, no calculations are necessary. Try to draw the Feynman diagrams for the two decays and justify why the second is so much smaller than the first; in fact, so small that it has not yet been observed).

**Problem 7**

Consider a large water Cherenkov detector for solar neutrinos. The electron neutrinos are detected by the reaction \( \nu_e + e^- \rightarrow \nu_e + e^- \). Assume the cross section, at about 10 MeV, \( \sigma = 10^{-47} \text{ m}^2 \) and the incident flux, in the range above threshold, \( \Phi = 10^{10} \text{ m}^{-2} \text{ s}^{-1} \). What is the water mass in which the interaction rate is 10 events a day if the detection efficiency is \( \varepsilon = 50\% \)? (Consider how many electrons there are per kilogram of water.)

**Problem 8**

Here we will try to explain why a pion decays almost exclusively into a muon even though there is much more phase space for the decay into the 200 times lighter electron: Calculate the ratio of the partial widths \( \Gamma (\pi^+ \rightarrow e^+ + \nu_e) \) and \( \Gamma (\pi^+ \rightarrow \mu^+ + \nu_e) \) following the steps described in class, but show all the intermediate steps in the calculation:

1. Calculate the ratio of the phase spaces for the two decays. Since this is a two-body decay, there is only one independent kinematic quantity, the momentum \( \vec{p} \) of either the charged lepton or the neutrino. The phase space density is proportional to \( dN/dE_0 \), where \( E_0 = m_\pi \) in the pion rest frame; since the pion has zero spin, the decay is isotropic and the number of states per unit volume in momentum is \( dN \propto p^2 dp \). Find
p in terms of $E_0$ in order to calculate $dN/dE_0$. Which decay mode is favored if only the phase space is considered?

2. Now consider that in weak interactions, the polarization of a lepton or antilepton is

$$\frac{N_1 - N_2}{N_1 + N_2} = \beta,$$

where $N_1$ is the number of leptons with the favored spin orientation (negative helicity, or left-handed, for leptons; positive, or right-handed, for antileptons) and $N_2$ is the number with the disfavored orientation (the opposite). Neutrinos are massless, or nearly so, therefore $\beta = 1$: the neutrinos are 100% polarized. From the momentum of the charged lepton, found in (1), calculate $\beta$ and from that the probability that the charged lepton will have the required spin orientation to conserve angular momentum. Use the ratio of these probabilities for $e^+$ and $\mu^+$ to modify the result in (1). Explain in words the reason the charged pion decays almost exclusively into muons, even though the electron is so much lighter.