Problem 1

Make a table as below and for each interaction type, strong (S), electromagnetic (EM), and Weak (W), write a Y or N in the appropriate cell for each quantum number, depending on whether it is conserved or not ($I$ isospin; $I_3$ isospin third component; $S$ strangeness; $B$ baryon number; $L$ lepton number; $T$ time reversal; $C$ particle-antiparticle conjugation; $P$ parity; $J$ angular momentum; $J_z$ angular momentum third component).

<table>
<thead>
<tr>
<th></th>
<th>$I$</th>
<th>$I_3$</th>
<th>$S$</th>
<th>$B$</th>
<th>$L$</th>
<th>$T$</th>
<th>$C$</th>
<th>$P$</th>
<th>$J$</th>
<th>$J_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td></td>
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<td></td>
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<tr>
<td>EM</td>
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<tr>
<td>W</td>
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</tr>
</tbody>
</table>

Problem 2

Antihyperons were first produced in 1958, in an experiment where a beam of negative pions with energy $E_\pi = 4.6 \text{ GeV}$ hit an emulsion stack. What is the final state containing a $\bar{\Lambda}$ that can be produced in a $\pi^- p$ collision with the lowest beam energy? (Which interaction is responsible for this process? Think about all the quantum numbers that must be conserved.) Find this threshold energy if the target protons are free and, approximately, if they are bound inside nuclei with a Fermi momentum $p_F = 150 \text{ MeV}$. Assuming that the pion beam was produced at a distance $l = 8 \text{ m}$ upstream of the emulsion and that the number of produced pions was $N_0 = 10^6/\text{cm}^2$, how many pions per cm$^2$ reached the emulsion?
Problem 3

For each of the following reactions establish whether it is allowed or not; if not, give the reason(s):

1. $\mu^+ \rightarrow e^+ + \gamma$;
2. $e^- \rightarrow v_e + \gamma$;
3. $p + p \rightarrow \Sigma^+ + K^+$;
4. $p + p \rightarrow p + \Sigma^+ + K^-$;
5. $p \rightarrow e^+ + \nu_e$;
6. $p + p \rightarrow \Lambda + \Sigma^+$;
7. $p + n \rightarrow \Lambda + \Sigma^+$;
8. $p + n \rightarrow \Xi^0 + p$;
9. $p \rightarrow n + e^+ + \nu_e$;
10. $n \rightarrow p + e^- + \nu_e$.

Problem 4

A $\pi^-$ is captured by a deuteron $d$ ($J^P = 1^-$) and produces the reaction $\pi^- + d \rightarrow n + n$.

a If the capture is from an $S$ state, what is the total spin of the two neutrons and what is their orbital angular momentum?

b Show that, if the capture is from a $P$ state, the neutrons are in a singlet state.

Problem 5

The positronium is an atomic system made by an $e^-$ and an $e^+$ bound by the electromagnetic force.

a Determine the relationship that this condition imposes between the orbital angular momentum $l$, the total spin $s$, and the charge conjugation $C$.

b Determine the relationship between $l$, $s$, and $N$ (the number of photons) which allows the reaction $e^- + e^+ \rightarrow N\gamma$ to occur without violating $C$.

c What is the minimum number of photons in which the ortho-positronium ($^3S_1$) and the para-positronium ($^1S_0$) can annihilate respectively?
Problem 6

An $\eta$ meson decays into $2\gamma$ while moving in the $x$ direction with energy $E_\eta = 5\text{ GeV}$.

1. If the two gammas are emitted in the $+x$ and $-x$ directions, what are their energies?

2. If the two gammas are emitted at equal and opposite angles $\pm \theta$ with the $x$ axis, what is the angle between the two?

Problem 7

Consider Table 3.1 in your textbook:

<table>
<thead>
<tr>
<th>$J^{PC}$</th>
<th>$1S_0$</th>
<th>$3S_1$</th>
<th>$1P_1$</th>
<th>$3P_0$</th>
<th>$3P_1$</th>
<th>$3P_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$0^{-+}$</td>
<td>$1^{--}$</td>
<td>$1^{+-}$</td>
<td>$0^{++}$</td>
<td>$1^{++}$</td>
<td>$2^{++}$</td>
</tr>
</tbody>
</table>

(a) Using the rules derived in class (and also given in the book), justify the $J^{PC}$ values for the states listed in the table.

(b) Show that a meson with quantum numbers $J^{PC} = 0^{+-}$ cannot be a $q\bar{q}$ state (a quark and its antiquark) by considering all the possible $L, S$ combinations.