Problem 1
Consider an electron beam of energy $E = 2 \text{GeV}$ hitting an iron target (assume it is made of pure $^{56}\text{Fe}$). How large is the maximum four-momentum transfer? (Consider which scattering angle results in maximum $Q^2$.)

Problem 2
Geiger and Marsden performed what is now known as the Rutherford experiment. They observed that alpha particles, after hitting a thin metal foil, not too infrequently bounced back. Calculate the ratio between the scattering probabilities for $\theta > 90^\circ$ and for $\theta > 10^\circ$.

Problem 3
Electrons with 10 GeV energy are scattered at 30$^\circ$ by protons initially at rest. Find the maximum energy of the scattered electrons.
Note: This is deep-inelastic scattering, so for a given scattering angle there is a whole range of energies transferred from the beam to the target (the cross section depends on two variables) and therefore a range of energies $E'$ for the scattered electrons. Find the condition under which $E'$ becomes maximum.

Problem 4
In a deep-inelastic scattering experiment aimed at studying the proton structure, an $E = 100 \text{GeV}$ electron beam hits a liquid hydrogen target. The energy $E'$ and the direction of the scattered electrons are measured. If $x$ and $Q^2$ are, respectively, the momentum fraction and the four-momentum transfer squared, find $E'$ for $Q^2 = 25 \text{GeV}^2$ and $x = 0.2$.

Problem 5
In the HERA collider an electron beam of energy $E_e = 30 \text{GeV}$ hits a proton beam with energy $E_p = 820 \text{GeV}$. The energy and the direction of the scattered electron are measured
in order to study the proton structure. Calculate the CM energy $\sqrt{s}$ and the energy $E_{e,f}$ an electron beam must have in order to reach the same $\sqrt{s}$ with a fixed proton target. Calculate the maximum four-momentum transfer of the electron $Q_{\text{max}}^2$ for $x = 0.4$, 0.01, and 0.0001. Hint: Be careful not to use expressions that were derived for the lab frame in the case where the proton target was at rest. You can safely use the relativistically invariant expression for any quantity.

**Problem 6**

Evaluate the ratio $\alpha/\alpha_s$ at $Q^2 = (10 \text{ GeV})^2$ and at $Q^2 = (100 \text{ GeV})^2$. Take $\Lambda_{\text{QCD}} = 200 \text{ MeV}$, $\alpha^{-1}(m_Z) = 129$, and $m_Z = 91 \text{ GeV}$.

**Problem 7**

Derive the following useful relationships and sum rules in the quark-parton model.

1. The quark-parton model predicts a simple relation between the structure functions of the nucleon (average of proton and neutron) $F_{2eN}(x)$ and $F_{2eN}^e(x)$ measured in neutrino-nucleon and electron-nucleon deep-inelastic scattering.

   (a) Derive this relation, $F_{2eN}(x) = c F_{2eN}^e(x)$ and find the numerical value of $c$ if strange quarks and antiquarks are neglected.

   (b) Give a modified form of (a) including $s(x)$ and $\bar{s}(x)$ and explain how it can be used to extract information on the strange content of the nucleon.

2. The Gottfried sum rule makes a prediction for the integral $\int_0^1 \frac{dx}{x} [F_{2eN}^p(x) - F_{2eN}^e(x)] = c^i$.

   (a) Derive the sum rule and find $c^i$, assuming that the light-quark sea is symmetric: $\bar{u}(x) = \bar{d}(x)$.

   (b) Relax the assumption of symmetry in (a) and derive a modified form for the sum rule that includes the term $\int_0^1 dx [\bar{d}(x) - \bar{u}(x)]$.

3. The Gross-Llewellyn Smith sum rule refers to an additional structure function, $F_3(x)$, that can be measured in neutrino-nucleon scattering and which is interpreted, in the quark-parton model, as $F_3(x) = [u(x) - \bar{u}(x)] - [d(x) - \bar{d}(x)]$. Derive the sum rule $\int_0^1 dx F_3(x) = c''$ and find the value of $c''$. 