Assignment 3
Solutions

Problem 1

a. We can apply the formula we derived in Problem 7 of the first homework. We have

\[ p_\mu = p_\nu = E_\nu = \frac{m_K^2 - m_\mu^2}{2m_K} = \frac{494^2 - 106^2}{2(494)} \text{ MeV} = 236 \text{ MeV}. \]

As for the energy of the muon, we can simply note that energy conservation in the \( K \) rest frame implies

\[ E_\mu = m_K - E_\nu = 258 \text{ MeV}. \]

b. We can apply a Lorentz transformation to the muon momentum from the kaon rest frame to the lab frame, but first we notice that the muons with maximum momentum are the ones emitted along the kaon direction of motion in the lab. The transformation for a boost parallel to the momentum is

\[ p^L_\mu = -\gamma \beta E^*_\mu + \gamma p^*_\mu, \]

where \( p^L_\mu \) is the muon momentum in the lab and quantities with stars are in the CM frame (kaon rest frame), calculated in part a. If \( \beta \) is the velocity of the kaon in the lab frame frame, \( \beta > 0 \) (the kaon is moving along the positive \( x \)-axis) then the velocity of the lab in the kaon rest frame, which is the quantity we need in the equation above, will be \( -\beta \). We can calculate \( \beta \) and \( \gamma \) from the kaon momentum and energy:

\[ \gamma/\beta = \frac{p_K}{m_K} = \frac{5000 \text{ MeV}}{494 \text{ MeV}} = 10.1 \]

and

\[ \gamma = \frac{\sqrt{p_K^2 + m_K^2}}{m_K} = \frac{\sqrt{(5000 \text{ MeV})^2 + (494 \text{ MeV})^2}}{494 \text{ MeV}} = \frac{5024 \text{ MeV}}{494 \text{ MeV}} = 10.2. \]

Then

\[ p^L_\mu = 10.1(258 \text{ MeV}) + 10.2(236 \text{ MeV}) = 5013 \text{ MeV}. \]
Problem 2

We need first to calculate the Lorentz factor:
\[ \gamma = \frac{E}{m}\pi = \frac{5\text{GeV}}{140\text{MeV}} = 35.7 \]

in order to calculate relativistic effects. In the pion rest frame, the distance to the Earth is Lorentz-contracted:
\[ L' = \frac{L}{\gamma} = \frac{30\text{km}}{35.7} = 840\text{ m}. \]

In the Earth frame, the pion lifetime is time-dilated, \( \tau' = \gamma \tau \), where \( \tau \) is the proper lifetime, and the distance traveled, from the perspective of an observer on Earth, within a lifetime will be
\[ d = c\gamma\tau = 35.7(3 \times 10^8 \text{ m/s})(26 \times 10^{-9} \text{ s}) = 278 \text{ m} \]
(at these energies, it is safe to take the velocity of the particle to be \( c \)).

Problem 3

Since the two particles are observed with the same radius of curvature in the chamber, their momenta must be equal. We use the practical formula (Eq. 1.68) giving the momentum of a particle in GeV if the field is in T and the radius in m:
\[ p = 0.3B\rho = 0.3 \times 0.2 \times 0.2 = 0.012 \text{ GeV}, \]

or \( p = 12 \text{ MeV} \) for each particle. Since the electron and the positron were created with zero angle between them, the momentum of the photon was simply the sum of the two momenta:
\[ p_\gamma = p_{e^+} + p_{e^-} = 24 \text{ MeV} \]
and so the photon energy was \( E = p_\gamma = 24 \text{ MeV} \).

Problem 4

1. \( \rho^0 \to \pi^+ + \pi^- \) Strong
2. \( K^+ \to \pi^0 + \pi^+ \) Weak
3. \( \eta^0 \to \pi^+ + \pi^- + \pi^0 \) Electromagnetic
4. \( \mu^- \to e^- + \bar{\nu}_e + \nu_\mu \) Weak
5. \( \pi^0 \to \gamma + \gamma \) Electromagnetic
Problem 5

The magnetic moment of an elementary fermion, such as a charged lepton, arising from its spin, is given by

\[ \mu = g \frac{q}{2m}s, \]

where \( q \) is the charge, \( m \) its mass, \( s \) the spin, and the \( g \)-factors are taken to be all \( \sim 2 \) (to a very good approximation). We see that it is inversely proportional to the mass, with all other factors being equal. Therefore,

\[ \frac{\mu_e}{\mu_{\mu}} = \left( \frac{0.511 \text{ MeV}}{106 \text{ MeV}} \right)^{-1} = 207 \]

and

\[ \frac{\mu_e}{\mu_{\tau}} = \left( \frac{0.511 \text{ MeV}}{1777 \text{ MeV}} \right)^{-1} = 3477. \]

Problem 6

When the target proton is not at rest, as a free proton would be, but rather has a motion due to its being inside a nucleus, where not all protons can be in the ground state, its momentum and kinetic energy contribute to the total center-of-mass energy \( \sqrt{s} \) available for producing particles in the final state. The calculation is the same. If 1 is the beam proton and 2 the target, we find

\[ s = (E_1 + E_2)^2 - (p_1 + p_2)^2 = E_1^2 + E_2^2 + 2E_1E_2 - p_1^2 - p_2^2 - 2p_1 \cdot p_2 = 2m_p^2 + 2E_1E_2 - 2p_1 \cdot p_2. \]

We see that this is maximized when the Fermi momentum of the target proton happens to be opposite to the beam direction, when it becomes

\[ s = 2m_p^2 + 2E_1E_2 + 2p_1p_2. \]

In order to produce an additional proton-antiproton pair in the final state, this must be at least equal to \( (4m_p)^2 \) (the center-of-mass energy of three protons and an antiproton at rest). So we must have

\[ E_1E_2 + p_1p_2 = 7m_p^2 \]

for the threshold energy \( E_1 \). Writing \( p_1 \approx E_1 \) and \( p_2 = p_F = 150 \text{ MeV} \), we can solve for \( E_1 \) to get

\[ E_1 = \frac{7m_p^2}{E_2 + p_F} = \frac{7m_p^2}{\sqrt{m_p^2 + p_F^2} + p_F} = \frac{7(0.938 \text{ GeV})^2}{\sqrt{(0.938 \text{ GeV})^2 + (0.15 \text{ GeV})^2} + 0.15 \text{ GeV}} = 5.6 \text{ GeV}. \]

This is the threshold proton beam energy for producing antiprotons on a heavy nucleus target. Note that could have simplified the expression above somewhat by writing \( E_2 \approx m_p \), since \( p_F^2 \ll m_p^2 \) (verify that this is a good approximation). Then the threshold energy would be written as

\[ E_1 \approx 7m_p \left( 1 - \frac{p_F}{m_p} \right); \]

the result should be about the same.
Problem 7

1. $M1$ serves to select negative particles of momentum $p = 1.19$ GeV, since knowing the momentum of the particles is crucial for the method of identifying them, as described later.

2. The quadrupole magnet focuses the selected particles so that they will pass through the first scintillation counter. Without focusing, the beam would be widely dispersed, since the particles come out of the production target at different angles.

3. The mass is determined by measuring both the momentum and the velocity of the particles.

4. The two methods for measuring the velocity were time-of-flight (time taken to traverse a fixed distance) and production of light in a Cherenkov detector sensitive only to velocities corresponding to antiprotons, as opposed to negative pions, with the above momentum. The reason two independent methods were employed was that the pion background was much larger than the searched antiproton signal (it is much easier to produce the lighter pions) and there was a high probability that two separate pions would give signals in the two counters forming the time-of-flight detector with (accidentally) the right timing to look like an antiproton.

5. $C1$ was set up to produce a signal if a negative pion with $p = 1.19$ GeV passed through, but not if an antiproton with the same momentum (and therefore much lower velocity) did. It was used as a “veto,” to reject particles that were obviously pions.